Time : 2 hours Max Marks : 40

GEOMETRIC GROUP THEORY - MID-SEMESTRAL EXAM.

Answer all questions. You may use results proved in class after correctly quoting them. Any other claim must be accompanied by a proof.

(1) Decide whether the following statements are *True* or *False*. Answers without correct and complete justifications will not be awarded any points.
  (a) $\langle a, b \,/\, a^{-1}bab^{-2}, b^{-1}aba^{-2} \rangle$ is a presentation of the trivial group.
  (b) If H is a finite index subgroup of a finitely generated group Γ, then H is finitely generated.
  (c) If $\Gamma = \langle X \,/\, R \rangle$ and f an automorphism of Γ, then the group
$$\langle X, a \,/\, R, a^{-1}xa = f(x), x \in X \rangle$$
surjects onto $\mathbb{Z}$.
  (d) Every geodesic metric space is convex.
  (e) The group $\mathcal{QI}(\mathbb{R})$ contains a free abelian group of infinite rank. [4x5=20]

(2) Let
$$\Gamma = \left\{ \frac{m}{2^n} \,/\, m, n \in \mathbb{Z} \right\}$$
Then Γ is a subgroup of the additive group of rationals. Show that
$$\langle b_1, b_2, \ldots \,/\, b_n = b_{n+1}^2, n = 1, 2, \ldots \rangle$$
is a presentation of Γ. Γ is called the group of dyadic rationals. [7]

(3) Define the term *quasi-isometry*. Show that any map which is within a bounded distance of a quasi-isometry is itself a quasi-isometry. [2+4]

(4) Show that the groups $\mathbb{Z}$ and $\mathbb{Z}_2 * \mathbb{Z}_2$ are quasi-isometric by explicitly constructing a quasi-isomtery between them. [7]